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Paper Code : 16 Sr. No. .....

MATHEMATICAL SCIENCES [Paper-II]

Signature and Name of Invigilator

1. (Signature) _____
   (Name) _____
2. (Signature) _____
   (Name) _____

OMR Sheet No. : .....
(To be filled by the candidate)

Roll No. | | | | | | | | | | |
(In Figures as per admission card)

Roll No. _____
(In words)

Time : 1¼ Hours] **[Maximum Marks : 100**

Number of Pages in this Booklet : 16 **Number of Questions in this Booklet : 50**

Instructions for the Candidates

1. Write your roll number in the space provided on the top of this page.
2. This paper consists of fifty multiple-choice type of questions.
3. At the commencement of examination, the question bookelt will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below :
   (i) Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Fault booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
   (ii) After this verification is over, the OMR Sheet Number should be entered on this Test Booklet.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the oval as indicated below on the correct response against each item.
   Example : (A) (B) ● (D)
   where (C) is the correct response.
5. Your responses to the items are to be indicated in the Answer Sheet given inside the Paper I Booklet only. If you mark at any place other than in the ovals in the Answer Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the test booklet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. You have to return the test question booklet and OMR Answer sheet to the invigilators at the end of the examination compulsorily and must not carry it with you outside the Examination Hall.
10. Students can take home carbon copy of this OMR answer sheet.
11. Use only Blue/Black Ball point pen.
12. Use of any calculator or log table etc., is prohibited.
13. There is no negative marks for incorrect answers.

परीक्षार्थियों के लिए निर्देश

1. पहले पृष्ठ के ऊपर नियत स्थान पर अपना रोल नम्बर लिखिए।
2. इस प्रश्न-पत्र में पचास बहुविकल्पीय प्रश्न हैं।
3. परीक्षा प्रारम्भ होने पर, प्रश्न-पुस्तिका आपको दे दी जायेगी। पहले पाँच मिनट आपको प्रश्न-पुस्तिका खोलने तथा उसकी निम्नलिखित जाँच के लिए दिये जायेंगे; जाँच की जाँच आपको अवश्य करनी है :
   (i) कवर पृष्ठ पर छपे निर्देशानुसार प्रश्न-पुस्तिका के पृष्ठ तथा प्रश्नों की संख्या को अच्छी तरह चैक कर लें कि ये पूरे हैं। दोषपूर्ण पुस्तिका जिनमें पृष्ठ/प्रश्न कम हों या दुबारा आ गये हों या सीरियल में न हों अर्थात् किसी भी प्रकार की त्रुटिपूर्ण पुस्तिका स्वीकार न करें तथा उसी समय उसे लौटाकर उसके स्थान पर दूसरी सही प्रश्न-पुस्तिका ले लें। इसके लिए आपको पाँच मिनट दिये जायेंगे। उसके बाद न तो आपकी प्रश्न-पुस्तिका वापस ली जायेगी और न ही आपको अतिरिक्त समय दिया जायेगा।
   (ii) इस जाँच के बाद OMR पत्रक की क्रम संख्या इस प्रश्न-पुस्तिका पर अंकित कर दें।
4. प्रत्येक प्रश्न के लिए चार उत्तर पत्रक विकल्प (A), (B), (C) तथा (D) दिये गये हैं। आपको सही उत्तर के दीर्घवृत्त को पेन से भरकर काला करना है जैसा कि नीचे दिखाया गया है।
   उदाहरण : (A) (B) ● (D)
   जबकि (C) सही उत्तर है।
5. प्रश्नों के उत्तर केवल प्रश्न पत्र I के अन्दर दिये गये उत्तर-पत्रक पर ही अंकित करने हैं। यदि आप उत्तर पत्रक पर दिये गये दीर्घवृत्त के अलावा किसी अन्य स्थान पर उत्तर चिह्नांकित करते हैं, तो उसका मूल्यांकन नहीं होगा।
6. अन्दर दिये गये निर्देशों को ध्यानपूर्वक पढ़ें।
7. कच्चा काम (Rough Work) इस पुस्तिका के अन्तिम पृष्ठ पर करें।
8. यदि आप उत्तर-पुस्तिका पर अपना नाम या ऐसा कोई भी निशान करते हैं तो परीक्षा के लिये अयोग्य घोषित कर दिये जायेंगे।
9. आपको परीक्षा समाप्त होने पर प्रश्न-पुस्तिका एवं OMR उत्तर-पत्रक निरीक्षक महोदय को लौटाना आवश्यक है और परीक्षा समाप्ति के बाद उसे अपने साथ परीक्षा भवन से बाहर न लेकर जायें।
10. परीक्षा समाप्ति पर परीक्षार्थी OMR उत्तर-पत्रक की कार्बन कापी अपने साथ ले जा सकते हैं।
11. केवल नीले/काले बाल प्वाईंट पेन का ही इस्तेमाल करें।
12. किसी भी प्रकार का संगणक (केलकुलेटर) या लाग टेबल आदि का प्रयोग वर्जित है।
13. गलत उत्तरों के लिए कोई अंक काटे नहीं जाएँगे।

Paper Code : [16]

Paper-II [MATHEMATICAL SCIENCES]

Note : • **This paper contains Fifty (50) multiple choice questions, each question carrying two (2) marks.**

नोट : • **इस प्रश्नपत्र में पचास (50) बहुविकल्पीय प्रश्न हैं। प्रत्येक प्रश्न के दो (2) अंक हैं।**

1. Let A and B be any two sets and A × B denote the cartesian product of A and B. Then :

   (A) If A × B is countable, then both A and B are countable.

   (B) If A × B is countable, then A is countable.

   (C) If A × B is countable, then B is countable.

   (D) If A × B is countable, then either A or B is countable.

2. One of the following statements is true. Identify :

   (A) Every subset of rational numbers which is bounded above has a unique supremum rational number.

   (B) Every subset of rational numbers which is bounded above has a unique supremum real number.

   (C) The set of rational numbers has least upper bound property.

   (D) The set of real numbers does not have the greatest lower bound property.

3. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by :

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$$

   Then at x = 0;

   (A) f is not continuous.

   (B) f is continuous but not differentiable.

   (C) f is differentiable but f' is not continuous.

   (D) f is differentiable and f' is continuous.

4. If f is an increasing function and g is a decreasing function on an interval such that fog exists, then :

(A) fog is an increasing function on I

(B) fog is decreasing function on I

(C) fog is neither increasing nor decreasing on I

(D) none of these

5. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = |x|^2$. Then :

(A) f is continuous and differentiable on $\to \mathbb{R}$ but is not uniformly continuous on $\mathbb{R}$.

(B) f is uniformly continuous on $\mathbb{R}$ but is not differentiable at 0.

(C) f is continuous but not differentiable or $\mathbb{R}$.

(D) f is uniformly Continuous $\mathbb{R}$.

6. Let $\{f_n\}$ be a sequence of functions, where $fn(x) = x^{-nx}$, $x \geq 0$. Then, the sequence $\{f_n\}$ converges :

(A) pointwise to a continuous function (B) uniformly on $[0, \infty]$

(C) uniformly on $(0, \infty)$ (D) uniformly on (k, ∞), whre $k > 0$

7. Let $f : [0, 1] \to R$ be a function such that |f| is Reimann integrable over [0, 1]. Which of the following is necessarily true ?

(A) f is Riemann integrable over [0, 1]

(B) f is Lebesgue integrable over [0, 1]

(C) f is both Riemann integerable and Lebesgue integrable over [a, b]

(D) f is neither Riemann integrable nor Lebesgue integrable

8. If $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & , \text{ if } (x,y) \neq (0,0) \\ 0 & , \text{ otherwise} \end{cases}$ then :

(A) f is continuous at (0, 0).

(B) f is not continuous at (0, 0) but $\lim_{(x,y)\to(0,0)} f(x,y)$ exist.

(C) $\lim_{(x,n)\to(0,0)} f(x,y)$ does not exist.

(D) Partial derivatives of f exist only at (0, 0).

9. Let $\{f_n\}$ be a sequence of functions, where fn : [0, 1] $\to$ $\mathbb{R}$ is defined by $f_n(x) = x^n$ for n = 1, 2, ... Then the sequence $\{f_n\}$ is :

(A) convergent but not uniformly convergent on [0, 1]

(B) uniformly convergent on [0, 1]

(C) not convergent on [0, 1]

(D) convergent to a continuous function on [0, 1].

10. Let α and β be two distinct real roots of a polynomial p(x). Then, there exists at least one root lying between α and β of the polynomial :

(A) p(x)

(B) p′ (x)

(C) p″ (x)

(D) none of the above

11. Let $K_1 = \left\{\frac{1}{n} : n = 1,2,3,.....\right\}, K_2 = K_1 \cup \{0\}$. Then :

(A) both K_1 and K_2 are compact subsets of $\mathbb{R}$. (B) K_1 is compact but not K_2 in $\mathbb{R}$.

(C) Neither K_1 nor K_2 is compact in $\mathbb{R}$. (D) K_2 is compact but K_1 is not compact in $\mathbb{R}$.

12. On the vector space R^2 define $\| \|_1$, $\| \|_2$, $\| \|_3$ and $\| \|_4$ as follows. For $x = (x_1, x_2) \in \mathbb{R}^2$, $\|x\|_1 = |x_1| + |x_2|$, $\|x\|_2 = x^2_1 + x^2_2$, $\|x\|_3 = \|x_1\|^3 + |x_2|^3$, $\|x\|_4 = \text{Max } \{|x_1|, |x_2|\}$. Then :

(A) only $\| \|_1$ and $\| \|_2$ are norms. (B) only $\| \|_1$ and $\| \|_3$ are norms.

(C) only $\| \|_1$ and $\| \|_4$ are norms. (D) only $\| \|_1$ and $\| \|_2$ are norms.

13. If U is a subspace of a vector space V then :

(A) $U + U \neq 2U$ (B) $U + U = U$

(C) $U + U = V$ (D) none of these

14. $A = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$ (a, b, c and d are reals) has an eigen value in R if and only if :

(A) $(a + d)^2 - 4bc \geq 0$ (B) $(a - d)^2 + 4bc \leq 0$

(C) $(a + d)^2 - 4bc \leq 0$ (D) $(a - d)^2 + 4bc \geq 0$

15. Let x, y be linearly independent vectors in R^2. Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(y) = \alpha x$ and $T(x) = 0$. Then with respect to some basis in $\mathbb{R}^2$, matrix of T is of the form :

(A) $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}, a > 0$ (B) $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, a \neq b$

(C) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

16. Suppose V is an inner product space and S is an isometry of V then :

(A) det (S) = 1 (B) |det (S)| = 1

(C) det (S) > 1 (D) det (S) < 1

17. If V is an inner product space with inner product given by <,> then $|\langle u, v\rangle| = \|u\| \|v\|$ for $u, v \in V$ iff :

(A) $u = \alpha v$ where α is an scalar (B) $v \neq \beta v$ where β is an scalar

(C) (A) & (B) (D) none of the above

18. Suppose T is a linear operator on a vector space V such that every vector in V is an eigen vector of T. Then :

(A) $T^2 = -T$ (B) $T = \alpha I$ for $\alpha \in \mathbb{R}$

(C) $T^2 = T$ (D) no such T exists

19. Let $q(x_1, x_2) = a x_1^2 + b x_1 x_2 + c x_2^2$ be the quadratic form associated with a symmetric bilinear form f on R^2. Then q is nongenerate if and only if :

(A) $b^2 - 4ac > 0$ (B) $b^2 - 4ac \neq 0$

(C) $b^2 - 4ac < 0$ (D) none of these

20. If W_1 and W_2 are finite - dimensional subspaces of a vector space V, then $W_1 + W_2$ is finite dimensional and :

(A) $\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$

(B) $\dim W_1 - \dim W_2 = \dim (W_1 \cap W_2) - \dim (W_1 + W_2)$

(C) $\dim W_1 + \dim W_2 = \dim W_2 = \dim (W_1 + W_2)$

(D) $\dim W_1 + \dim W_2 = \dim (W_1 + W_2) - \dim (W_1 \cap W_2)$

21. If R is the radius of consequence of the power series $\sum_{n=1}^{\infty} z^2 / n$ then :

(A) R = 0 (B) R < 1

(C) R = 1 (D) R > 1

22. Consider the integral :

$$I = \int_r \frac{e^z}{z^3} dz$$

where r is the circle centred at 0 and with radius 1. The the value of I is :

(A) $e^{\pi i}$ (B) πi

(C) $-\pi i$ (D) $\bar{e}^{\pi i}$

23. The function f defined by :

$$f(z) = \begin{cases} \frac{z^5}{|z|^4} & , \ z \neq 0 \\ 0 & , \ z = 0 \end{cases}$$

(A) satisfies CR equations at z = 0 and is also differentiable at z = 0

(B) satisfies CR equations at z = 0 but is not differentiable at z = 0

(C) does not satisfy CR equations at z = 0 and also is not differentiable at z = 0

(D) does not satisfy CR equating at z = 0 but is differentiable at z = 0

24. A Möbius transformation maps a straight line into :

(A) a straight line

(B) a circle

(C) a straight line or a circle

(D) a circle or an ellipse

25. Number of ring homomorphism from Z_{12} to Z_{30} is :

(A) 4

(B) 6

(C) 8

(D) 2

26. The characteristic of z[i]/ <2 + i> is :

(A) 0

(B) 5

(C) 3

(D) 2

27. Let ϕ be Euler's ϕ-function. m, n $\in \mathbb{Z}$, s.t. m and n have no common factor. Then :

(A) $\phi(m\,n) = \phi(m) + \phi(n)$

(B) $\phi(m\,n) = \phi(m)\,\phi(n)$

(C) $\phi(m\,n) = \phi(m) - \phi(n)$

(D) $\phi(m\,n) = \phi(m) + \phi(n) + \phi(m)\,\phi(n)$

28. Smallest natural number whose $\phi(n) = 8$:

(A) 15

(B) 16

(C) 24

(D) 25

29. For $n \geq 1$:

(A) There are at least $n + 1$ primes less than 2^n.

(B) There are at most $n + 1$ primes less than 2^n.

(C) There are at least $n - 1$ primes less than 2^n.

(D) There are at most $n - 1$ primes less than 2^n.

30. Let X = {a, b, c}, which one of the following is not true ?

(A) $\{\phi, x\}$ is a topology on x

(B) $\{x, \phi, \{a, b\}, \{b\}, \{b, c\}\}$ is a topology on x

(C) $\{x, \phi, \{b\}, \{a, b\}, \{b, c\}, \{c\}\}$ is a topology on x

(D) $\{\phi, \{a\}\}$ is a topology on x.

31. The solution of the initial value problem :

$$\frac{dy}{dx} = \frac{3x-2y}{x}; y(-5) = 3$$

is :

(A) $\left(x^3 - 200\right)/x^2$

(B) $\left(x^3 + 200\right)/x^2$

(C) $\left(x^3 + 50\right)/x^2$

(D) $\left(x^3 - 50\right)/x^2$

32. The solution of the differential equation :

$4xy\,dy + (3y^2 - 1)\,dx = 0$

is :

(A) $(3y^2 - 1)x^3 = c^3$

(B) $(3y^2 - 1)^2 x^3 = \pm e^{3c}$

(C) $(3y^2 - 1)^2 x^3 = c^3$

(D) $(3y^2 - 1)^2 x^3 = \log c$

33. The only solution of

$$y^{(n)} + P_1(x)y^{(n-1)} + \ldots\ldots + P_n(x)y = 0,$$

where the $P_i(x)\ (i = 1,2,\ldots.n)$ are continuous functions and

$$y(a) = y'(a) = \ldots\ldots = y^{(n-1)}(a) = 0$$

is

(A) $y \neq 0$ (B) $\frac{dy}{dx} \neq 0$

(C) $y \equiv 0$ (D) $\frac{dy}{dx} \equiv 0$

34. The complete integral of partial differential equation

zpq = p + q, where the symbols have their usual meanings, is :

(A) z = (1 + a) [x + ay] + b, a and b being arbitrary constants

(B) $z = 2(1+a)\left[x+\left(\frac{1}{a}\right)y\right]+b$, a and b being arbitrary constants

(C) $z^2 = (1+a)\left[x+\left(\frac{1}{a}\right)y\right]+b$, a and b being arbitrary constants

(D) $z^2 = 2(1+a)\left[x+\left(\frac{1}{a}\right)y\right]+b$, a and b being arbitrary constants

35. Interative formula for $\sqrt{N}$ is :

(A) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{1}{x_n}\right)$

(B) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$

(C) $x_{n+1} = \left(x_n - \frac{N}{x_n}\right)$

(D) None of these

36. Select the most appropriate choice in the following statements :

(A) More the order of convergence, more is the number of iteration

(B) More the order of convergence, less is the number of iteration

(C) Less the order of convergence, more is the number of iteration

(D) Less the order of convergence, less is the number of iteration

37. The curve through (1, 0) and (2, 1) which minimizes $\int_1^2 \{(1 + y'^2 / x^2)\}^{1/2}\, dx$ is:

(A) a circle

(B) a parabola

(C) an ellipse

(D) a hyperbola

38. Non-linear equations of the type

$$y(x) = \int_a^b k(x,t)\, \phi\, [t, y(t)]\, dt$$

are called :

(A) Convolution type equation

(B) Hammerstein type equation

(C) Fredholm equation of first kind

(D) Fredholm equation of second kind

39. Hamiltonian of a charged particle in an electromagnetic field is :

(A) $\frac{1}{2m}(p-qA)^2+q\phi$

(B) $\frac{1}{4m}(p+qA)^2$

(C) $\frac{1}{m}(p-qA)^2-q\phi$

(D) $\frac{1}{4m}(p-qA)+q\phi$

40. Which of the following is correct ?

(A) The generalized momentum increment is not equal to the generalized impulsive force associated with each generalized coordinate.

(B) The generalized momentum increment is equal to the generalized impulsive face associated with each generalized co-ordinate.

(C) The generalized momentum increment is not equal to the generalized impulsive couple associated with each generalized co-ordinate.

(D) None of these

41. For a random variable X, E(X) = 3 and V(X) = 4. The mean and standard deviation of Y = 3X – 2 is :

(A) 7, 6

(B) 7, 36

(C) 7, 10

(D) 7, 34

42. The total number of possible outcomes in tossing three dice simultaneously is :

(A) 6×3

(B) 3^6

(C) 6^3

(D) ${}^6C_3$

43. If A and B are two independent events such that $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{5}$, then $P(A/A\cup B)$ is :

(A) $\frac{5}{7}$ (B) $\frac{5}{6}$

(C) $\frac{5}{17}$ (D) Information is insufficient

44. If b_{yx} and b_{xy} are two regression coefficients and r is the correlation coefficient between :

X and Y then $\left|\frac{1}{2}(b_{xy}+b_{xy})\right|$ is :

(A) = r (B) = r^2

(C) < |r| (D) > |r|

45. Let $x_1, x_2 \ldots\ldots x_n$ be a random sample from a distribution wtih p.df f(x, θ). A statistic T = $t(x_1 \ldots\ldots x_n)$ is said to be sufficient for θ if the conditional distribution of $x_1 \ldots\ldots x_n$ given T = t is :

(A) Independent of θ (B) Independent of t

(C) Dependent on θ (D) Unbiased for θ

46. In which of the following designs, the principle of local control is **not** used :

(A) Completely Randomized Deisign (B) Randomized Block Design

(C) Latin Square Design (D) All of the above

47. The moment generating function of a random variable x is given by $M_x(t)e^{8t^2}$. The mode and standard deviation of x are :

(A) 0, 16 (B) 0, 4

(C) 2, 16 (D) 2, 4

48. Let π_i be the probability of inclusion of the i^{th} unit in a sample of size n from a population of size N. For a simple random sampling with replacement (SRSWR), the value of π_i is :

(A) $\frac{1}{N_{C_n}}$ (B) $\frac{n}{N}$

(C) $1-\frac{n}{N}$ (D) $\frac{n(n-1)}{N(N-1)}$

49. A new drug is tested for its effectiveness in reducing blood pressure. For this purpose, blood pressure of patients before the start of treatment and after treatment for a period of one month were noted. Which is the most appropriate parametric test for testing the effectiveness of the drug ?

(A) Paired t test (B) z test

(C) Chi square test (D) Wilcoxon signed rank test

50. Let X be a random variable with E(X) = 1 and V(X) = 4 then $P(-3 \leq X \leq 5)$ is :

(A) ≥ 0.75 (B) ≤ 0.75

(C) = 0.75 (D) Information is insufficient