CCE-(P) - 2015
MATHEMATICS

KTM–20–XV

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Subject Code : 2 0

Test Booklet No. : 00920

TEST BOOKLET

MATHEMATICS

Time Allowed : 2 (Two) Hours **Full Marks : 200**

INSTRUCTIONS

1. The name of the Subject, Roll Number as mentioned in the Admission Certificate, Test Booklet No. and Subject Code shall be written legibly and correctly in the space provided on the Answer Sheet with black ball pen.
2. **Space provided for Series in the Answer Sheet is not applicable for Optional Subject. So the space shall be left blank.**
3. All questions carry equal marks. Your total marks will depend only on the number of correct responses marked by you in the Answer Sheet.
4. No candidate shall be admitted to the Examination Hall/Room 20 minutes after commencement of distribution of the paper. The Supervisor of the Examination Hall/Room will be the time-keeper and his/her decision in this regard is final.
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8. Complete silence must be observed in the Examination Hall/Room. No candidate shall copy from the paper of any other candidate, or permit his/her own paper to be copied, or give, or attempt to give, or obtain, or attempt to obtain irregular assistance of any kind.
9. After you have completed filling in all your responses on the Answer Sheet and the Examination has concluded, you should hand over to the Invigilator *only the Answer Sheet*. You are permitted to take away with you the Test Booklet.
10. Violation of any of the above Rules will render the candidate liable to expulsion from the Examination Hall/Room and disqualification from the Examination, and according to the nature and gravity of his/her offence, he/she may be debarred from future Examinations and Interviews conducted by the Commission for appointment to Government Service.
11. Smoking inside the Examination Hall/Room is strictly prohibited.
12. **This Test Booklet contains three pages for Rough Work at the end.**

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[No. of Questions : 100]

SEAL

CCE (P) - 2015
MATHEMATICS

1. For any two sets A and B, $A \cap (A \cup B)'$ is equal to

(A) A (B) B

(C) ϕ (D) $A \cap B$

2. If A and B be two non-empty sets, then which one of the following is the result of $(A \cup B) - A$?

(A) $A - B$ (B) $B - A$

(C) B (D) A'

3. Consider the function $f : W \to W$ defined by

$$f(n) = \begin{cases} n+1 \text{ if } n \text{ is even} \\ n-1 \text{ if } n \text{ is odd} \end{cases}$$

where W is the set of whole numbers. Then the function f is

(A) one-one

(B) onto

(C) bisective

(D) None of the above

4. If A and B are two sets such that $n(A) = 2$, $n(B) = 3$ then $n(P(A \times B))$ is equal to

(A) 64 (B) 6

(C) 5 (D) 12

5. With respect to composite composition, the set A_n of all even permutations of degree n, defined on a set S forms a finite group of order

(A) n (B) $\angle n$

(C) $n/2$ (D) $\angle n/2$

6. If in a group of 65 people, 40 like cricket, 10 like both cricket and tennis, then the number of people who like tennis only and not cricket is

(A) 15 (B) 25

(C) 35 (D) 45

7. If in a group G, $a^5 = e$, the identity element of G and $aba^{-1} = b^2$ for $a, b \in G$ then $O(b)$ is

(A) 31 (B) 29

(C) 23 (D) 19

8. The set Q of rational numbers is not a group under the operation $*$ defined by $a * b = \frac{ab}{2}$ for

(A) $*$ is not a binary operation in Q

(B) $*$ is not associative in Q

(C) there does not exist identity element in Q

(D) some elements of Q have no inverses

9. Let H and K be two subgroups of a group G. Then $H \cup K$ is a subgroup of G if and only if

(A) either $H \subseteq K$ or $K \subseteq H$

(B) $H \cap K = \phi$

(C) $H \not\subseteq K$ and $K \not\subseteq H$

(D) None of the above

10. Let $(Q, +, \cdot)$ be a ring of rational numbers. Then the set of integers Z is

(A) a subring but not an ideal of Q

(B) neither a subring nor an ideal of Q

(C) a subring as well as an ideal of Q

(D) not a subring but an ideal of Q

11. The inverse of a symmetric matrix is

(A) symmetric

(B) skew-symmetric

(C) diagonal matrix

(D) None of the above

12. The rank of a $m \times n$ matrix whose every element is unity is

(A) m (B) n

(C) mn (D) 1

13. The system of linear equations $AX = B$ is consistent if and only if the coefficient matrix A and the augmented matrix $[AB]$ are of

(A) same rank

(B) different ranks

(C) same number of rows

(D) None of the above

14. If $x, y, z \in R^+$ and $x^3 + y^3 + z^3 = 81$ then the maximum value of $x + y + z$ is

(A) $3\sqrt{3}$ (B) 3

(C) 27 (D) 9

15. The condition in order that the equation $x^3 - px^2 + qx - r = 0$ may have a pair of equal roots is

(A) $pq = r$

(B) $pr = q$

(C) $qr = p$

(D) None of the above

16. The infinite series

$$\frac{1+2}{2^3} + \frac{1+2+3}{3^3} + \frac{1+2+3+4}{4^3} + \cdots\infty$$

is

(A) convergent

(B) oscillatory

(C) divergent

(D) None of the above

17. The series

$$\frac{1}{2}+\frac{4}{9}x+\frac{9}{28}x^2+\cdots+\frac{n^2}{n^3+1}x^{n-1}+\cdots$$

is convergent if

(A) $x<1$

(B) $x>1$

(C) $x=1$

(D) None of the above

18. Choose the incorrect statement :

(A) A convergent sequence determines its limit uniquely.

(B) Every convergent series is bounded.

(C) A monotonic increasing sequence diverges to $+\infty$ if it is not bounded above.

(D) A monotonic decreasing sequence is not convergent.

19. $\lim\limits_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)$ is equal to

(A) 0 (B) $\frac{1}{2}$

(C) log 2 (D) e^4

20. The function f defined by

$$f(x)=\begin{cases}\dfrac{3|x|+4\tan x}{x}, & x\neq 0\\ k, & x=0\end{cases}$$

is continuous at $x=0$ for

(A) $k=0$

(B) $k=1$

(C) $k=7$

(D) no value of k

21. If $y=\log(\sin x)$, then y_3 is equal to

(A) $\dfrac{\sin x+\cos^2 x}{\sin^4 x}$

(B) $\dfrac{2\cos x}{\sin^3 x}$

(C) $\dfrac{1+\cos^2 x}{\sin^3 x}$

(D) $\dfrac{\sin^2 x+2\cos x}{\sin^3 x}$

22. Suppose the function f satisfies the conditions

(i) $f(x+y)=f(x)f(y)\ \forall x$ and y

(ii) $f(x)=1+xg(x)$ where

$$\lim_{x\to 0} g(x)=1$$

Then $f'(x)$ is equal to

(A) $f(x)$

(B) $f(x)\cdot g(0)$

(C) $2f(x)\cdot g(0)$

(D) None of the above

23. If $\phi(x,\ y)=x^3 y\sin^{-1}\left(\dfrac{y}{x}\right)$ and

$$x\frac{\partial\phi}{\partial x}+y\frac{\partial\phi}{\partial y}=n\phi$$

then n is equal to

(A) 4 (B) 5

(C) 6 (D) 7

24. If $u = x^2y + y^2z + z^2x$ then the value of $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ is equal to

(A) 0

(B) u

(C) $x + y + z$

(D) $(x + y + z)^2$

25. The point of intersection of the tangents to the curve $y = 2x^2$ at the points (1, 2) and (−1, 2) is

(A) (0, 0) (B) (0, 2)

(C) (0, −2) (D) (2, 0)

26. If H is a homogeneous function of x and y of degree k, then each of $\frac{\partial H}{\partial x}$ and $\frac{\partial H}{\partial y}$ is a homogeneous function in x and y of degree

(A) k

(B) $k - 1$

(C) $k - 2$

(D) None of the above

27. The function $f(x, y) = 2x^2 + 2xy - y^3$ has

(A) no stationary point

(B) only one stationary point at (0, 0)

(C) two stationary points at (0, 0) and $\left(\frac{1}{6}, -\frac{1}{3}\right)$

(D) two stationary points at (0, 0) and (1, −1)

28. $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$ is equal to

(A) 0

(B) $\frac{3}{2}$

(C) 1

(D) −1

29. If $F(x) = \int \frac{x + \sin x}{1 + \cos x} dx$ and $F(0) = 0$, then $F(\pi/2)$ is equal to

(A) $\pi/2$

(B) $\pi/3$

(C) $\pi/4$

(D) π

30. The value of $\int_0^1 \frac{\log(1+x)}{1+x} dx$ is

(A) $\frac{1}{5}\log 2$

(B) $\log 2$

(C) $\frac{\pi}{8}\log 2$

(D) $\pi \log 2$

31. The volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about the x-axis is

(A) $\frac{4}{3}\pi a^2 b$

(B) $\frac{4}{3}\pi ab^2$

(C) $\frac{2}{3}\pi a^2 b$

(D) $\frac{2}{3}\pi ab^2$

32. The triangle of maximum area which can be inscribed in a circle is

(A) scalene

(B) right angled

(C) isosceles

(D) equilateral

33. If $f(x) = f(a-x)$, $g(x) + g(a-x) = 4$ and $\int_0^a f(x)dx = \frac{1}{2}$, then $\int_0^a f(x)g(x)\,dx$ is equal to

(A) 1

(B) 0

(C) -1

(D) None of the above

34. The radius of curvature of the parabola $y^2 = 4x$ at the vertex $(0, 0)$ is equal to

(A) 0 (B) 1

(C) 2 (D) 3

35. The value of $\int \sin^{-1}(\cos x)\,dx$ is

(A) $\frac{\pi x}{2} - \frac{x^2}{2} + C$

(B) $\frac{\pi x^2}{2} - \frac{x}{2} + C$

(C) $\frac{\pi x}{2} + C$

(D) None of the above

36. In the mean value theorem

$$f(x+h) = f(x) + hf'(x+\theta h),\ 0 < \theta < 1$$

where $f(x) = \sin x$, the limiting value of θ as $h \to 0+$ is

(A) 2 (B) $\frac{1}{2}$

(C) 1 (D) $-\frac{1}{2}$

37. If $f(x) = \int_0^{x^2} \sqrt{t}\,dt$, then $\frac{df(x)}{dx}$ is equal to

(A) 0 (B) 1

(C) $2x^2$ (D) $\sqrt{x}$

38. If at every point of a certain curve, the slope of the tangent equals $-\frac{2x}{y}$, then the curve is

(A) a straight line

(B) a circle

(C) a parabola

(D) an ellipse

39. If

$$\int_0^1 \cot^{-1}(1 - x + x^2)\,dx = \lambda \int_0^1 \tan^{-1} x\,dx$$

then the value of λ is equal to

(A) 1 (B) 2

(C) 3 (D) 4

40. The condition $f'(c) = 0$ for $f(c)$ to be an extreme value of the function f is

(A) necessary condition

(B) sufficient condition

(C) necessary and sufficient conditions

(D) necessary but not sufficient condition

41. The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$$

is

(A) 3

(B) 2

(C) 1

(D) not defined

42. The integrating factor of the differential equation

$$y\,dx - (x + 2y^2)\,dy = 0$$

is

(A) $1/y$

(B) $1/x$

(C) y

(D) None of the above

43. $(y - cx)(c - 1) = c$ is a solution of the differential equation

(A) $y = px + p - p^2$; $p = \dfrac{dy}{dx}$

(B) $y = px - \dfrac{ap^2}{p+1}$

(C) $y = px + \dfrac{p}{p-1}$

(D) None of the above

44. The particular integral of the differential equation

$$(D^3 - D)y = e^x + e^{-x}$$

is given by

(A) $\frac{1}{2}(e^x + e^{-x})$

(B) $\frac{1}{2}x(e^x + e^{-x})$

(C) $\frac{1}{2}x^2(e^x + e^{-x})$

(D) $\frac{1}{2}x^2(e^x - e^{-x})$

45. The differential equation

$$M(x, y)dx + N(x, y)dy = 0$$

is an exact equation if

(A) $\dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} = 0$

(B) $\dfrac{\partial M}{\partial y} + \dfrac{\partial N}{\partial x} = 0$

(C) $\dfrac{\partial M}{\partial x} - \dfrac{\partial N}{\partial y} = 0$

(D) None of the above

46. The general solution of the differential equation

$$(D^2 + 6D + 9)y = 2e^{-2x}$$

is

(A) $y = (A\cos 3x + B\sin 3x) + 2e^{-2x}$

(B) $y = cxe^{-3x} + 2e^{-2x}$

(C) $y = (c_1 + c_2 x)e^{-3x} + e^{-2x}$

(D) $y = (c_1 + c_2 x)e^{-3x} + 2e^{-2x}$

47. If the straight lines

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

intersect on the x-axis then

(A) $ag = fh$

(B) $ah = fg$

(C) $af = gh$

(D) None of the above

48. If by transformation from one rectangular axes to another with the same origin, the expression $ax + by$ changes to $a'x' + b'y'$, then

(A) $a + b = a' + b'$

(B) $a^2 + b^2 = a'^2 + b'^2$

(C) $\dfrac{a}{a'} + \dfrac{b}{b'} = 1$

(D) $\dfrac{a}{a'} = \dfrac{b}{b'}$

49. The equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of perpendicular lines if

(A) $a + b = 0$

(B) $h^2 = ab$

(C) $h^2 > ab$

(D) $h^2 < ab$

50. If the shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$

and $$\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

is $\lambda\sqrt{30}$ unit, then the value of λ is

(A) 1

(B) 2

(C) 3

(D) 4

51. Reducing to its standard form, the equation

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$$

transforms to

(A) $y^2 = 24x$

(B) $\dfrac{x^2}{4} - \dfrac{y^2}{6} = 1$

(C) $\dfrac{y^2}{6} - \dfrac{x^2}{4} = 1$

(D) $\dfrac{x^2}{4} + \dfrac{y^2}{6} = 1$

52. The direction cosines of the line of shortest distance between the lines

$$\frac{x+3}{-4} = \frac{y-6}{3} = \frac{z}{2}$$

and $\frac{x+2}{-4} = \frac{y}{1} = \frac{z-7}{1}$

are

(A) (1/9, –4/9, 8/9)

(B) $(1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$

(C) (1/9, 4/9, 8/9)

(D) $(1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14})$

53. The number of spheres of a given radius r and touching the coordinate axes is

(A) 4

(B) 6

(C) 8

(D) None of the above

54. The equation $xy = 0$ in three-dimensional space represents

(A) a pair of straight lines

(B) a plane

(C) a pair of parallel lines

(D) a pair of planes at right angles

55. The cone

$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$

has three mutually perpendicular generators if

(A) $a + b + c = 1$

(B) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 1$

(C) $a + b + c = 0$

(D) None of the above

56. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually non-coplanar mutually perpendicular unit vectors then $[\vec{a} \ \vec{b} \ \vec{c}]$ is

(A) 0 (B) 1

(C) 2 (D) 3

57. The vector function $\vec{f}(z)$ of the scalar variable t is of constant direction if

(A) $\vec{f} \cdot \frac{d\vec{f}}{dt} = 0$

(B) $\vec{f} \times \frac{d\vec{f}}{dt} = \vec{0}$

(C) $\frac{d\vec{f}}{dt} = \vec{0}$

(D) None of the above

58. If the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} - 3\hat{k}$ and $3\hat{i} + a\hat{j} + 5\hat{k}$ are coplanar, then the value of a is

(A) –4

(B) 4

(C) 0

(D) None of the above

59. The area of the parallelogram whose adjacent sides are $\hat{i}+\hat{k}$ and $2\hat{i}+\hat{j}+\hat{k}$ is given by

(A) $\sqrt{2}$

(B) $\sqrt{3}$

(C) 3

(D) $2\sqrt{3}$

60. The general value of $(-i)^{-i}$ is

(A) $e^{(4n+1)\frac{\pi}{2}}$

(B) $e^{-(4n+1)\frac{\pi}{2}}$

(C) $e^{(1-4n)\frac{\pi}{2}}$

(D) $e^{(4n-1)\frac{\pi}{2}}$

61. If $a=\cos\alpha+i\sin\alpha$, $b=\cos\beta+i\sin\beta$, $c=\cos\gamma+i\sin\gamma$, then $abc+\frac{1}{abc}$ is equal to

(A) 0

(B) $2\cos(\alpha+\beta+\gamma)$

(C) $2\sin(\alpha+\beta+\gamma)$

(D) $\tan(\alpha+\beta+\gamma)$

62. If $\tan(x+iy)=u+iv$, then the value of $u^2+v^2+2u\cot 2x$ is

(A) 0

(B) -1

(C) 1

(D) 2

63. If α is a positive acute angle, then the imaginary part of $\log(1+i\tan\alpha)$ is

(A) α

(B) $\tan\alpha$

(C) $\sec\alpha$

(D) 0

64. Which of the following is incorrect?

(A) $\cosh^2 x-\sinh^2 x=1$

(B) $\text{sech}^2 x-\tanh^2 x=1$

(C) $\text{sech}^2 x+\tanh^2 x=1$

(D) $\coth^2 x-\text{cosech}^2 x=1$

65. The sum of the series

$$\left(\frac{2}{3}+\frac{1}{7}\right)-\frac{1}{3}\left(\frac{2}{3^3}+\frac{1}{7^3}\right)+\frac{1}{5}\left(\frac{2}{3^5}+\frac{1}{7^5}\right)-\cdots$$

is given as

(A) 0

(B) π

(C) $\pi/2$

(D) $\pi/4$

66. If a function f is invertible, then the number of inverses of f can be

(A) 2

(B) 3

(C) infinite

(D) 1 i.e., unique

67. The domain of the function

$$f(x)=\frac{1}{\sqrt{x-5}}$$

is given as

(A) $]5, \infty[$

(B) $[5, \infty[$

(C) $]-\infty, \infty[$

(D) $]-\infty, 5]$

68. The total number of injective mapping from a set with m-elements to a set with n-elements when $m \le n$ is

(A) m^n

(B) n^m

(C) $\frac{\lfloor n}{\lfloor n-m}$

(D) $\lfloor n$

69. A subgroup H of a group G is a normal subgroup if and only if

(A) $xH = x^{-1}H \quad \forall x \in G$

(B) $xHx^{-1} = H \quad \forall x \in G$

(C) $h^{-1}xh \in G \quad \forall h \in H$

(D) $hG = Gh \quad \forall h \in H$

70. Let $(G, \cdot)$ be a group and $a \in G$. If $O(a) = 40$ then $O(a^{15})$ is

(A) 120

(B) 5

(C) 8

(D) 80

71. Let f be a homomorphism from a group G into a group G' with $\ker f = \{e\}$. Then f is

(A) an epimorphism

(B) a monomorphism

(C) an isomorphism

(D) None of the above

72. An integral domain is

(A) a field when it is finite

(B) always a field

(C) never a field

(D) None of the above

73. If A is a singular matrix then Adj A is

(A) non-singular

(B) singular

(C) symmetric

(D) Not defined

74. If A and B are symmetric matrices of the same order, then

(A) $A - B$ is a skew-symmetric matrix

(B) $AB - BA$ is a symmetric matrix

(C) $AB + BA$ is a symmetric matrix

(D) AB is a symmetric matrix

75. If A and B are square matrices of same order and A' denotes the transpose of A, then

(A) $(AB)' = A' + B'$

(B) $(AB)' = A'B'$

(C) $(AB)' = B'A'$

(D) $(AB)' = A' - B'$

76. If X be an invertible matrix, then X^{-1} is

(A) $\dfrac{\text{Adj } X}{|X|}$

(B) $|X| \text{ Adj } X$

(C) $X\,(\text{Adj } X)$

(D) None of the above

77. The condition which must be satisfied by the coefficients of the equation

$$x^3 - px^2 + qx - r = 0$$

when two of its roots α, β are connected by a relation $\alpha + \beta = 0$, is

(A) $pr + q = 0$

(B) $pq - r = 0$

(C) $pq + r = 0$

(D) $p - qr = 0$

78. If α, β, γ be the roots of the equation $x^3 + 6x^2 + 3x + 1 = 0$, then the value of $\alpha^2 + \beta^2 + \gamma^2$ is

(A) 26

(B) 30

(C) 36

(D) 24

79. The condition that the roots of the equation $ax^3 + bx^2 + cx + d = 0$ may be geometrical progression is

(A) $ac = bd$

(B) $ad = bc$

(C) $ac^3 = b^3 d$

(D) $a^3 c = b^3 d$

80. The second term of the equation

$$x^3 - 3x^2 + 12x + 16 = 0$$

can be removed if we diminish the roots by

(A) 4

(B) 3

(C) 2

(D) 1

81. If $\lim_{n \to \infty} \frac{u_{n+1}}{u_n} = k$, then the positive term series Σu_n converges if

(A) $k > 1$

(B) $k < 1$

(C) $k = 1$

(D) None of the above

82. The series, whose nth term is $\sqrt{n^2 - 1} - n$, is

(A) oscillatory

(B) convergent

(C) divergent

(D) None of the above

83. The value of ξ in the mean value theorem $f(b)-f(a)=(b-a)f'(\xi)$ for $f(x)=\sqrt{x}$ and $a=4$, $b=9$, is

(A) $\frac{3}{2}$

(B) $\frac{25}{4}$

(C) $1-\sqrt{\frac{7}{12}}$

(D) None of the above

84. Rolle's theorem is not applicable to the function $f(x)=|\sin x|$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ because

(A) $f\left(-\frac{\pi}{2}\right) \neq f\left(\frac{\pi}{2}\right)$

(B) f is not continuous in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(C) f is not derivable at some points of $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(D) f is not defined at some points of $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

85. If the function $f(x)=(x-3)^2$ satisfies all the conditions of Lagrange's mean value theorem, then the point on the curve where the tangent is parallel to the chord joining the points (3, 0) and (4, 1) is

(A) (1, 4)

(B) (0, 9)

(C) (5/2, 1/4)

(D) (7/2, 1/4)

86. If $u=\log(x^3+y^3+z^3-3xyz)$, then $u_x+u_y+u_z$ is equal to

(A) $\frac{3}{x+y+z}$

(B) $\frac{1}{x+y+z}$

(C) $\frac{x^2+y^2+z^2}{x+y+z}$

(D) $\frac{x+y+z}{x^2+y^2+z^2}$

87. The angle of intersection of the curves $r=a(1+\cos\theta)$, $r=b(1-\cos\theta)$ is

(A) π

(B) $\pi/2$

(C) 0

(D) $-\pi/2$

88. The maximum value of $\sin x+\cos x$ is

(A) 2

(B) $\sqrt{2}$

(C) 1

(D) $1+\sqrt{2}$

89. The value of $\int_0^1 \tan^{-1} \frac{2x-1}{1+x-x^2} dx$ is

(A) 1

(B) 0

(C) −1

(D) $\pi/4$

90. The value of the indefinite integral

$$\int \frac{dx}{3+2\sin x+\cos x}$$

is given by

(A) $\tan^{-1}\left(1+\tan\frac{x}{2}\right)+C$

(B) $\sin^{-1}\left(1+\tan\frac{x}{2}\right)+C$

(C) $\cos^{-1}\left(1+\tan\frac{x}{2}\right)+C$

(D) $\tan^{-1}\left(1+\cos\frac{x}{2}\right)+C$

91. The polar equation of a conic is

$$\frac{5}{r} = 3-4\cos\theta$$

Then the eccentricity of this conic is given by

(A) 4/3

(B) 5/3

(C) 3/4

(D) 5/4

92. The area of the region bounded by the parabola $y^2 = x$ and the line $2y = x$ is

(A) 1 sq. unit

(B) $\frac{1}{3}$ sq. unit

(C) $\frac{2}{3}$ sq. unit

(D) $\frac{4}{3}$ sq. unit

93. The slope of the tangent at (x, y) to a curve passing through the point (2, 1) is $\frac{x^2+y^2}{2xy}$. Then the equation of the curve is given by

(A) $2(x^2-y^2) = 3x$

(B) $2(x^2-y^2) = 3y$

(C) $x(x^2-y^2) = 6$

(D) $x(x^2+y^2) = 10$

94. The general solution of the differential equation

$$\frac{d^2y}{dx^2} = e^{-2x}$$

is given by

(A) $y = \frac{1}{4}e^{-2x} + ax + b$

(B) $y = \frac{1}{4}e^{-2x} + a$

(C) $y = e^{-2x} + ax + b$

(D) $y = \frac{1}{4}e^{-2x} + ax^2 + b$

95. The solution of the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^x$$

is given by

(A) $e^x(c_1 + c_2x) + x^2e^x$

(B) $e^x(c_1 + c_2x) + \frac{1}{3}x^3e^x$

(C) $e^x(c_1 + c_2x) + \frac{1}{12}x^4e^x$

(D) None of the above

96. The value of λ for which the two circles $x^2 + y^2 + 5x + 3y + 7 = 0$ and $x^2 + y^2 - 8x + 6y + \lambda = 0$ are orthogonal, is

(A) 18

(B) -18

(C) 9

(D) 0

97. The number of normals which can be drawn from a given point to a parabola is

(A) 1

(B) 2

(C) 3

(D) 6

98. For any three vectors $\vec{a}, \vec{b}, \vec{c}$; the value of $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}]$ is equal to

(A) $2[\vec{a}\ \vec{b}\ \vec{c}]$

(B) $\frac{1}{2}[\vec{a}\ \vec{b}\ \vec{c}]$

(C) $[\vec{a}\ \vec{b}\ \vec{c}]^2$

(D) None of the above

99. If $\alpha = \cos x + i\sin x$, $\beta = \cos y + i\sin y$, $\gamma = \cos z + i\sin z$ and $\alpha + \beta + \gamma = 0$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ is

(A) $3\cos(x+y+z)$

(B) $3\sin(x+y+z)$

(C) 0

(D) None of the above

100. If $a = \cos\theta + i\sin\theta$, $b = \cos\phi + i\sin\phi$, then the value of $\cos(\theta + \phi)$ is

(A) $\frac{1}{2}\left(ab - \frac{1}{ab}\right)$

(B) $\frac{1}{2}\left(ab + \frac{1}{ab}\right)$

(C) $\frac{a^2 + b^2}{2ab}$

(D) $\frac{a^2 - b^2}{2ab}$