2018

PHYSICS - PAPER-I

Full Marks : 200

Time Allowed : 3 Hours

If the questions attempted are in excess of the prescribed number, only the questions attempted first up to the prescribed number shall be valued and the remaining ones ignored.

Answers may be given either in English or in Bengali but all answers must be in one and the same language.

## GROUP - A

Answer any three questions

1. (a) Write the Maxwell's relations of thermodynamics.

   (b) Consider entropy to be a function of T and V.

   Show that

   $$Tds = C_v dT + T\left(\frac{\partial P}{\partial T}\right)_V dV$$

   The symbols have their usual meaning.

   (c) Now, take the entropy to be a function of T and P.

   Show that $Tds = CpdT - T\left(\frac{\partial V}{\partial T}\right)_p dP$

   The symbols have their usual meaning.

   (d) An ideal gas with pressure $P_1$, volume $V_1$ and temperature T is expanded isothermally to pressure $P_2$, volume $V_2$. Find the work done.

   (e) A diatomic gas ($\gamma = 1.4$) does 200J of work when it is expanded isobarically. Find the heat given to the gas in the process.

   (f) Briefly describe the operation of a Carnot cycle with the help of a P-V diagram and find its efficiency.

   8+6+6+5+5+10

2. (a) State Gauss's Law of electrostatics. Find both integral and differential forms of it.

   (b) Use Gauss's Law to find the field inside a charged sphere.

   (c) Find the potential due to a ring of charges at a point on the axis of the ring.

   (d) Show that the potential due to an electric dipole is

   $$\phi = \frac{\vec{p}.\vec{r}}{4\pi\epsilon_0 |\vec{r}|^3}$$ where the symbols have their usual meaning.

   Use this expression to find the components of electric field in spherical polar coordinates.

   (3+4+4)+5+8+(10+6)

3. (a) Set up the Lagrangian of a simple pendulum and obtain its equation of motion.

   (b) Suppose Hamiltonian of a system is

   $H = \sum_{i=1}^{N} p_i q_i - L$ (where the symbols have their usual meaning)

   Find the Hamilton's equations -

   (i) When H does not contain time explicitly

   (ii) When H contains time explicitly.

(c) A particle moves in the x-y plane under the influence of a central force depending only on the distance from the origin.

(i) Set up the Hamiltonian of the system.

(ii) Write Hamilton's equation of motion. (7+5)+(8+8)+(6+6)

4. (a) State the basic postulates of special theory of relativity.

(b) Write down the Lorentz transformation relations.

(c) Use the Lorentz transformation relations to find expressions for time-dialation and length contraction.

(d) Find the relativistic velocity addition formula.

(e) Define covariant and contravariant vectors.

(f) Define space-like and time-like vectors.

4+6+(5+5)+6+(4+4)+(3+3)

5. (a) Two masses $m_1$ and $m_2$ travelling in the same straight line collide. Find the velocities of the particles after collision in terms of the velocities before collision.

(Take 'e' to be the coefficient of restitution)

(b) Discuss the above problem for the case -

(i) perfectly elastic collision

(ii) perpectly inelastic collision.

(c) Show that for a perfectly elastic collision of the particles the total kinetic energy before collision is equal to the total kinetic energy after collision.

(d) Consider a square plate of side 'a'. Find

(i) moments of inertia about an axis comuting with any edge.

(ii) products of inertia.

(iii) principal moments of inertia. 8+(3+3)+6+(6+6+8)

6. (a) For an A.C. given by $i = i_0 \sin(\omega t + \phi)$. Show that $i_{rms} = i_0/\sqrt{2}$.

(b) An A.C. circuit contains only a capacitor. Find the relation between peak e.m.f. and peak current. Hence find the reactance.

(c) State Faraday's and Lenz's Law of electromagnetic induction.

(d) A D.C. battery is connected in a circuit containing an inductor (with inductance 'L') and a resistor (with resistance 'R') in series. The e.m.f. of the battery is 'E'. Find the expression of current in the circuit after a time 't'.

(e) Explain self induction. Find the self inductance of a solenoid of length '$\ell$', number of turns per unit length 'n' and radius 'r'.

7+7+(3+3)+10+(3+7)

## GROUP - B

Answer any two questions

7. (a) State and explain Biot-Savart's Law. Use it to find the magnetic field, due to current in a long straight wire, at a distance 'd' from it.

(b) Write the Maxwell's equations of electromagnetic theory, in the medium.

(c) Suppose a point charge '$q$' is held at a distance 'd' from an infinite grounded conducting plane. Use the method of images to find (i) the potential, (ii) the induced surface charge density and (iii) the total induced charge.

(d) From the Maxwell's equations find the wave equations for electric and magnetic field. (5+10)+6+(3+5+4)+7

8. (a) At time t = 0 a parachutist having weight 'mg' is at z=0 and travelling vertically downward with speed '$v_0$'. If the force on air resistance acting on the parachute is proportional to the instantaneous speed find the (i) speed, (ii) distance travelled and (iii) accelaration ; at any time $t > 0$. (iv) Find also the limiting speed.

(b) Consider two simple harmonic motions perpendicular to each other given as

$$x = a \sin \omega t \text{ and } y = b \sin(\omega t - \alpha)$$

(i) Show that the resultant motion is elliptic

(ii) Find the direction of the major axis of the ellipse.

(iii) Find the motion for $\alpha = 0$ and $\alpha = \pi/2$. (8+6+3+3)+(10+5+5)

9. (a) For a particle moving in a central force field

(i) Show that the motion is in a plane
(ii) Show that the angular momentum is conserved.
(iii) Write the equation of motion.
(iv) Show that the force field is conservative.

(b) Consider a vector $\vec{A}$ observed in a fixed coordinate system and also in another coordinate system rotating with angular velocity $\vec{\omega}$ which is constant in time.

(i) Show that $\left.\frac{d\vec{A}}{dt}\right|_{fix} = \left.\frac{d\vec{A}}{dt}\right|_{rot} + \vec{\omega} \times \vec{A}$

(ii) Use the above expression to show that

$$\left.\frac{d^2\vec{r}}{dt^2}\right|_{fix} = \left.\frac{d^2\vec{r}}{dt^2}\right|_{rot} + 2\vec{\omega} \times \left.\frac{d\vec{r}}{dt}\right|_{rot} + \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

(where $\vec{r}$ is the position vector) (6+4+6+4)+(10+10)

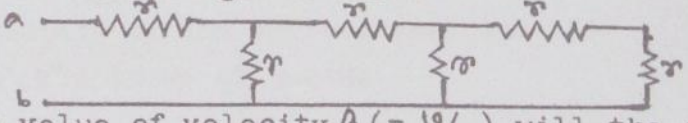
10. (a) Consider a source of sound (with velocity 'c') in relative motion with the observer. The original frequency is 'n'. Find the frequency measured by the observer when

(i) the source is moving with a velocity '$v$'

(ii) the observer is moving with a velocity '$v$'.

(b) Define extensive and intensive thermodynamic quantities. Give examples.

(c) Find the equivalent resistance between the points 'a' and 'b' for the circuit given below -




(d) For what value of velocity $\beta (= v/c)$ will the relativistic mass of a particle exceed its rest mass by a fraction 'f'.

(e) Two mercury drops each of radius 'r' merge to form a bigger drop. Calculate the surface energy released if 'S' is the surface tension.

(5+5)+(3+3+2)+7+7+8