

DO NOT OPEN THIS TEST BOOKLET UNTIL YOU ARE ASKED TO DO SO

**COMBINED COMPETITIVE (PRELIMINARY) EXAMINATION, 2013**

Serial No.

**MATHEMATICS**

**Code No. 13**



*Time Allowed : Two Hours*

*Maximum Marks : 300*

**INSTRUCTIONS**

1. IMMEDIATELY AFTER THE COMMENCEMENT OF THE EXAMINATION, YOU SHOULD CHECK THAT THIS TEST BOOKLET DOES NOT HAVE ANY UNPRINTED OR TORN OR MISSING PAGES OR ITEMS, ETC. IF SO, GET IT REPLACED BY A COMPLETE TEST BOOKLET.
2. ENCODE CLEARLY THE TEST BOOKLET SERIES **A, B, C OR D** AS THE CASE MAY BE IN THE APPROPRIATE PLACE IN THE RESPONSE SHEET.
3. You have to enter your Roll Number on this Test Booklet in the Box provided alongside.  
*DO NOT* write *anything else* on the Test Booklet.
4. This Booklet contains 100 items (questions). Each item comprises *four* responses (answers). You will select *one* response which you want to mark on the Response Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.
5. In case you find any discrepancy in this test booklet in any question(s) or the Responses, a written representation explaining the details of such alleged discrepancy, be submitted within three days, indicating the Question No(s) and the Test Booklet Series, in which the discrepancy is alleged. Representation not received within time shall not be entertained at all.
6. You have to mark all your responses **ONLY** on the separate Response Sheet provided. *See directions in the Response Sheet.*
7. All items carry equal marks. Attempt **ALL** items. Your total marks will depend only on the number of correct responses marked by you in the Response Sheet.
8. Before you proceed to mark in the Response Sheet the response to various items in the Test Booklet, you have to fill in some particulars in the Response Sheet as per instructions sent to you with your Admit Card and Instructions.
9. While writing Centre, Subject and Roll No. on the top of the Response Sheet in appropriate boxes use **“ONLY BALL POINT PEN”**.
10. After you have completed filling in all your responses on the Response Sheet and the examination has concluded, you should hand over to the Invigilator only the Response Sheet. You are permitted to take away with you the Test Booklet.

Your Roll No.

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## ROUGH WORK

1. If  $A = \{x, y \mid x^2 + y^2 = 25\}$  and  $B = \{x, y \mid x^2 + 9y^2 = 144\}$  then  $A \cap B$  contains :  
 (A) One point (B) Two points  
 (C) Three points (D) Four points
2. The number of subsets of a set containing  $n$  elements is :  
 (A)  $n$  (B)  $2^n - 1$   
 (C)  $n^2$  (D)  $2^n$
3. 20 teachers of a school either teach Maths or Physics. 12 of them teach Maths while 4 teach both the subjects. The number of teachers teaching Physics only is :  
 (A) 12 (B) 8  
 (C) 16 (D) None of these
4. If a relation  $R$  is defined on the set  $Z$  of integers as follows : . Then  
 Domain  $(R) =$   
 (A)  $\{3, 4, 5\}$  (B)  $\{0, 3, 4, 5\}$   
 (C)  $\{0, \pm 3, \pm 4, \pm 5\}$  (D) None of these
5. If  $R$  is a relation on a finite set having  $n$  elements, then the number of relations on  $A$  is :  
 (A)  $2^n$  (B)  
 (C)  $n^2$  (D)  $n^n$
6.  $R$  is a relation on the set  $Z$  of integers and it is given by . Then  $R$  is :  
 $(x, y) \in R \Leftrightarrow x^2 + y^2 = 1$   
 (A) Reflexive and Transitive (B) Reflexive and Symmetric  
 (C) Symmetric and Transitive (D) An equivalence relation
7. The equation represents a circle of radius :  
 (A) 5 (B)  $2\sqrt{5}$   
 (C)  $\frac{5}{2}$  (D) None of these
8. If  $Z_1, Z_2, Z_3$  are complex numbers such that :  
 $|Z_1| = |Z_2| = |Z_3| = \left| \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} \right| = 1$  then  $|Z_1 + Z_2 + Z_3|$  is :  
 (A) Equal to 1 (B) Less than 1  
 (C) Greater than 1 (D) Equal to 3
9. The locus of point  $Z$  satisfying  $\operatorname{Re}(Z^2) = 0$  is :  
 (A) A pair of straight lines (B) A circle  
 (C) A rectangular hyperbola (D) None of these

10. If  $Z_r = \cos\left(\frac{2r\pi}{5}\right) + i \sin\left(\frac{2r\pi}{5}\right)$ ,  $r = 0, 1, 2, 3, 4$  then  $Z_0 \times Z_1 \times Z_2 \times Z_3 \times Z_4$
- (A) -1 (B) 0  
(C) 1 (D) None of these
11. If  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 + 4x + 1 = 0$ . Then  $(\alpha + \beta)^{-1} + (\beta + \gamma)^{-1} + (\gamma + \alpha)^{-1} =$
- (A) 2 (B) 3  
(C) 4 (D) 5
12. Let A, G and H be the Arithmetic mean, Geometric mean and Harmonic mean of two positive numbers a and b. The quadratic equation whose roots are A and H is :
- (A)  $Ax^2 - (A^2 + G^2)x + AG^2 = 0$  (B)  $Ax^2 - (A^2 + H^2)x + AH^2 = 0$   
(C)  $Hx^2 - (H^2 + G^2)x + HG^2 = 0$  (D) None of these
13. G is a group under  $\otimes_7$  where  $G = \{1, 2, 3, 4, 5, 6\}$ . If  $5 \otimes_7 x = 4$  then  $x =$
- (A) 0.8 (B) 4  
(C) 3 (D) 5
14. In the group  $G = \{1, 3, 7, 9\}$  under multiplication module 10,  $(3 \times 7^{-1})^{-1}$  is equal to :
- (A) 9 (B) 5  
(C) 7 (D) 3
15. The identity element in the group  $M = \left\{ \begin{pmatrix} x & x \\ x & x \end{pmatrix} \mid x \neq 0 \text{ and } x \text{ is real} \right\}$  with respect to matrix multiplication is :
- (A)  $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$  (B)  $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$   
(C)  $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$  (D) None of these
16. If  $a * b = a^2 + b^2$ , then the value of  $(4 * 5) * 3$  is :
- (A)  $(4^2 + 5^2) + 3^2$  (B)  $(4 + 5)^2 + 3^2$   
(C)  $41^2 + 3^2$  (D)  $(4 + 5 + 3)^2$

17. In  $Z$ , the set of all integers, the inverse of -7 with respect to  $\oplus$  defined by  $a \oplus b = a + b + 1$  for all  $a, b \in Z$  is :
- (A) -14 (B) 7  
(C) -7 (D) None of these
18. The units of the field  $F = \{0, 2, 4, 6, 8\}$  under  $\oplus$  are :
- (A)  $\{0\}$  (B)  $\{2, 4, 6, 8\}$   
(C)  $F$  (D) None of these
19.  $(Z_n, \oplus_n, \otimes_n)$  is a field if and only if  $n$  is :
- (A) Even (B) Odd  
(C) Prime (D) None of these
20. The ideals of a field  $F$  are :
- (A) Only  $\{0\}$  (B) Only  $F$   
(C) Both  $\{0\}$  and  $F$  (D) None of these
21. Every finite integral domain is :
- (A) Not a field (B) Field  
(C) Vector space (D) None of these
22. The order of  $i$  in the multiplicative group of fourth roots of unity is :
- (A) 4 (B) 3  
(C) 2 (D) 1
23. The non-zero elements  $a, b$  of a ring  $(R, +, \cdot)$  are called zero divisors if :
- (A)  $a \cdot b = 0$  (B)  $a + b = 0$   
(C)  $a \cdot b = 1$  (D)  $a + b = 1$
24. If the ring  $R$  is an integral domain then :
- (A)  $R[x]$  is a field (B)  $R[x]$  is an integral domain  
(C)  $R[x]$  is not an integral domain (D) None of these
25. The product of an even permutation and an odd permutation is :
- (A) Even (B) Odd  
(C) Neither even nor odd (D) None of these

26. If \_\_\_\_\_ :

(A)  $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

(B)  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(C)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D) None of the above

27. If  $AB = A$  and  $BA = B$  where  $A$  and  $B$  are square matrices then :

(A)  $A^2 = A$  and  $B^2 = B$

(B)  $A^2 \neq A$  and  $B^2 = B$

(C)  $A^2 = A$  and  $B^2 \neq B$

(D)  $A^2 \neq A$  and  $B^2 \neq B$

28. If  $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ , then the value of  $|\text{adj } A|$  is :

(A)  $a^{27}$

(B)  $a^9$

(C)  $a^6$

(D)  $a^2$

29. If  $A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ , then  $|\text{adj}(\text{adj } A)|$  is :

(A)  $14^4$

(B)  $14^3$

(C)  $14^2$

(D)  $14$

30. If  $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$ , and  $A^T + A = I_2$  where  $A^T$  is the transpose of  $A$  and  $I_2$  is the  $2 \times 2$  Unit matrix.

Then:

(A)  $\theta = n\pi, n \in \mathbb{Z}$

(B)

(C)  $\theta = 2n\pi + \frac{\pi}{3}, n \in \mathbb{Z}$

(D) None of these

31. The matrix  $A = \begin{bmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{bmatrix}$  is nilpotent of index :
- (A) 2 (B) 3  
(C) 4 (D) None of these

32. The rank of the matrix  $A = \begin{bmatrix} 2 & 3 & 1 & 4 \\ 0 & 1 & 2 & -1 \\ 0 & -2 & -4 & 2 \end{bmatrix}$  is :
- (A) 2 (B) 3  
(C) 1 (D) Indeterminate

33. For what value of  $\lambda$ , the system of equations  
 $x + y + z = 6$   
 $x + 2y + 3z = 10$   
 $x + 2y + \lambda z = 12$  is Inconsistent ?
- (A)  $\lambda = 1$  (B)  $\lambda = 2$   
(C)  $\lambda = -2$  (D)  $\lambda = 3$

34. If A is a  $3 \times 3$  matrix and B is its adjoint such that  $|B| = 64$ , then  $|A| =$
- (A) 64 (B)  $\pm 64$   
(C)  $A + A^2 + A^3 + \dots \infty$  (D) 18

35. If  $A^3 = 0$ , then  $1 + A + A^2$  equals :
- (A)  $1 - A$  (B)  $(1 - A)^{-1}$   
(C)  $(1 + A)^{-1}$  (D) None of these

36. If  $A =$  equals to :
- (A)  $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$  (B)  $\begin{bmatrix} -1 & -2 \\ -3 & -4 \end{bmatrix}$   
(C)  $\begin{bmatrix} \frac{1}{2} & -\frac{1}{3} \\ -\frac{1}{2} & 0 \end{bmatrix}$  (D)  $\begin{bmatrix} -\frac{1}{4} & \frac{1}{3} \\ \frac{1}{2} & 0 \end{bmatrix}$

37. If  $s = a + b + c$  then the value of  $\Delta = \begin{vmatrix} s+c & a & b \\ c & s+a & b \\ c & a & s+b \end{vmatrix}$  is :

- (A)  $2s^2$  (B)  $2s^3$   
 (C)  $s^3$  (D)  $3s^3$

38.  $\lim_{n \rightarrow \infty} \left[ \frac{4^{\frac{1}{n}} - 1}{3^{\frac{1}{n}} - 1} \right]$  is equal to :

- (A)  $\log_4 3$  (B)  $\log_3 4$   
 (C) 1 (D) None of these

39. The value of  $\lim_{n \rightarrow \infty} \left[ \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n+1)(2n+3)} \right]$  is :

- (A) 1 (B)  $\frac{1}{2}$   
 (C)  $-\frac{1}{2}$  (D) None of these

40.  $\lim_{x \rightarrow \infty} \left[ \frac{\int_0^{2x} xe^{x^2} dx}{e^{4x^2}} \right] =$

- (A) 0 (B)  $\infty$   
 (C) 2 (D)

41. The function  $f(x) = \begin{cases} 1 - 2x + 3x^2 - 4x^3 + \dots + \infty & \text{if } x \neq -1 \\ 1 & \text{if } x = -1 \end{cases}$  is :

- (A) Continuous and differentiable at  $x = -1$   
 (B) Neither continuous nor differentiable at  $x = -1$   
 (C) Continuous but not differentiable at  $x = -1$   
 (D) None of the above

42. Let  $f(x) = \begin{cases} \frac{\sin \pi x}{5x} & , x \neq 0 \\ K & , x = 0. \end{cases}$

If  $f(x)$  is continuous at  $x = 0$ , then the value of  $K$  is :

- (A)  $\frac{\pi}{5}$  (B)   
 (C) 1 (D) 0

43. If  $f(x)$  is differentiable and strictly increasing function, then the value of  $\lim_{x \rightarrow 0} \left[ \frac{f(x^2) - f(x)}{f(x) - f(0)} \right]$  is :

- (A) 1 (B) 0   
 (C) -1 (D) 2

44. The number of points at which the function  $f(x) = |x - 3| + |x + 1|$  does not have a derivative in the interval  $[-4, 4]$  is :

- (A) 1 (B) 2   
 (C) 3 (D) None of these

45. If  $f(x)$  satisfies the conditions of Rolle's theorem in  $[1, 2]$  and  $f(x)$  is continuous in  $[1, 2]$ , then

$\int_1^2 f'(x) dx$  is equal to :

- (A) 3 (B) 0   
 (C) 1 (D) 2

46. Let  $f(x) = e^x$ ,  $x \in [0, 1]$ , then a number 'c' of the Lagrange's mean value theorem is :

- (A)  $\log_e (e - 1)$  (B)  $\log_e (e + 1)$    
 (C) 1 (D) None of these

47. The maximum value of  $xy$  subject to  $x + y = 8$  is :

- (A) 8 (B) 16   
 (C) 20 (D) 24

48. The series  $n - \frac{n^2}{2} + \frac{n^3}{3} - \frac{n^4}{4} + \dots - 1 < n \leq 1$  represents the function :

- (A)  $\sin n$  (B)  $\cos n$    
 (C)  $(1 + n)^n$  (D)  $\log (1 + n)$

49. Expansion of  $\sin x$  in powers of  $\left(x - \frac{\pi}{2}\right)$  is :

(A)  $\left(x - \frac{\pi}{2}\right) - \frac{\left(x - \frac{\pi}{2}\right)^3}{\underline{3}} + \frac{\left(x - \frac{\pi}{2}\right)^5}{\underline{5}} - + \dots$

(B)  $\left(x - \frac{\pi}{2}\right) + \frac{\left(x - \frac{\pi}{2}\right)^3}{\underline{3}} + \frac{\left(x - \frac{\pi}{2}\right)^5}{\underline{5}} + \dots$

(C)  $1 - \frac{\left(x - \frac{\pi}{2}\right)^2}{\underline{2}} + \frac{\left(x - \frac{\pi}{2}\right)^4}{\underline{4}} - + \dots$

(D) None of these

50. The equation of tangent to the curve  $x = t^3 - 4$ ,  $y = 2t^2 + 1$  at the point where  $t = 2$  is :

(A)  $2x - 3y - 19 = 0$

(B)  $2x - 3y + 19 = 0$

(C)  $2x + 3y - 19 = 0$

(D)  $3x + 2y + 6 = 0$

51. If the normal to the curve  $y^2 = 5x - 1$  at the point  $(1, -2)$  is of the form  $ax - 5y + b = 0$ . Then 'a' and 'b' are :

(A) 4, -14

(B) 4, 14

(C) -4, 14

(D) -4, -14

52. The least value of  $f(x) = 2x + \frac{8}{x^2}$ ,  $x > 0$  is :

(A) 4

(B) 6

(C) 8

(D) None of these

53. The radius of curvature for the curve  $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} - \frac{r^2}{a^2 b^2}$  is :

(A)  $\frac{p^2}{a^2 b^2}$

(B)  $\frac{a^2 p^2}{b^2}$

(C)  $\frac{a^2 b^2}{p^3}$

(D)  $a^2 b^2 p^2$

54. The centre of curvature of the curve  $y = x^2$  at  $(0,0)$  is :
- (A)  $\left(0, \frac{1}{2}\right)$  (B)  $\left(\frac{1}{2}, \frac{1}{2}\right)$   
 (C)  $\left(\frac{1}{2}, 0\right)$  (D) None of these
55. The radius of curvature of the curve  $r = a \sin n \theta$  at origin is :
- (A)  $na$  (B)  $\frac{2na}{3}$   
 (C)  $2an$  (D)  $\frac{2na}{3}$
56. The asymptote parallel to co-ordinate axes of the curve  $(x^2 + y^2)x - ay^2 = 0$  is :
- (A)  $y - a = 0$  (B)  $y + a = 0$   
 (C)  $x - a = 0$  (D)  $x + a = 0$
57. The asymptote of the curve  $y = e^x$  is given by :
- (A)  $y = 0$  (B)  $x = 0$   
 (C)  $y = e$  (D)  $x = e$
58. For the curve  $y^2(1+x) = x^2(1-x)$ , the origin is a :
- (A) Node (B) Cusp  
 (C) Conjugate point (D) None of these
59. The curve  $y = x^3 - 3x^2 - 9x + 9$  has a point of inflexion at :
- (A)  $x = -1$  (B)  $x = 1$   
 (C)  $x = -3$  (D)  $x = 3$
60. The curve  $y = \log x$  is :
- (A) Concave upwards in  $(0, \infty)$  (B) Concave downwards in  $(0, \infty)$   
 (C) Concave upwards in  $(-\infty, \infty)$  (D) Concave downwards in  $(-\infty, \infty)$
61. The points of inflexion on the curve  $x = (\log y)^3$  are :
- (A)  $(0, 1)$  and  $(8, e^2)$  (B)  $(1, 0)$  and  $(8, e^2)$   
 (C)  $(0, 1)$  and  $(e^2, 8)$  (D)  $(1, 0)$  and  $(e^2, 8)$
62. The graph of  $x = \frac{1-t^2}{1+t^2}$ ,  $y = \frac{2t}{1+t^2}$  is a :
- (A) Circle (B) Ellipse  
 (C) Cycloid (D) None of these

63. The number of leaves in the curve  $r = a \sin 5\theta$  are :
- (A) Two (B) Five  
(C) Ten (D) None of these
64. If  $u = f(y+ax) + \phi(y-ax)$  then  $\frac{\partial^2 u}{\partial x^2} =$
- (A)  $\frac{\partial^2 u}{\partial y^2}$  (B)  $a^2 \frac{\partial^2 u}{\partial y^2}$   
(C)  $-a^2 \frac{\partial^2 u}{\partial y^2}$  (D)  $a \frac{\partial^2 u}{\partial y^2}$
65. If  $Z = \log(x^2 + y^2)$  then  $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} =$
- (A) 0 (B) 1  
(C) 2 (D) 3
66. If  $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \dots + \infty}}}$  then  $(2y-1) \frac{dy}{dx}$  is given by :
- (A)  $\sin x$  (B)  $\cos x$   
(C)  $\tan x$  (D)  $\cot x$
67. The series  $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - + \dots$  is :
- (A) Conditionally Convergent (B) Absolutely Convergent  
(C) Divergent (D) None of the above
68. The series  $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \dots$  is :
- (A) Conditionally Convergent (B) Absolutely Convergent  
(C) Oscillatory (D) None of the above
69. The series  $\sum_{n=1}^{\infty} \frac{(n - 2 \log n)^n}{2^n n^n}$  is :
- (A) Convergent (B) Divergent  
(C) Oscillatory (D) None of these

70. The series  $\sum_{n=1}^{\infty} \frac{n 2^n}{n^n}$  is :

- (A) Convergent (B) Divergent  
(C) Oscillatory (D) None of these

71. The series  $\sum_{n=1}^{\infty} \frac{4 \cdot 7 \cdot \dots \cdot (3x+1)}{1 \cdot 2 \cdot \dots \cdot x} x^n$  is Convergent if :

- (A)  $|x| < 1$  (B)  
(C)  $|x| < \frac{1}{4}$  (D)  $|x| < \frac{1}{2}$

72.  $\int_0^2 \frac{\sqrt{x}}{\sqrt{3-x} + \sqrt{x}} dx =$

- (A) 0 (B)  $\frac{1}{2}$   
(C) 1 (D) None of these

73.  $\int_0^{\frac{\pi}{2}} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}} dx =$

- (A)  $\left[ \frac{1}{3n+1} + \frac{1}{2} \frac{1}{n+3} + \dots + \frac{1}{2n} \right] =$  (B)  
(C) (D)

74.

- (A)  $\log_e 2$  (B)  $\log_e 3$   
(C)  $\log_e 6$  (D) None of these

75. The entire length of the curve  $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$  is :

- (A)  $8a$  (B)  $4\sqrt{3}a$   
(C)  $6a$  (D)  $\sqrt{8a}$

76. The perimeter of  $r = a(1 + \cos\theta)$  is :  
 (A)  $a$  (B)  $2a$   
 (C)  $4a$  (D)  $8a$
77. The length of one arch of Cycloid  $x = a(\theta + \sin\theta)$   $y = a(1 - \cos\theta)$  is :  
 (A)  $a$  (B)  $4a$   
 (C)  $8a$  (D)  $32a$
78. The area bounded by the curve  $y = 2x$ ,  $x$  - axis and the ordinates  $x = -2$ ,  $x = 3$  is equal to :  
 (A)  $2$  (B)  $13$   
 (C)  $4$  (D)  $8$
79. The area of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is :  
 (A)  $2\pi ab$  (B)  $\pi ab$   
 (C)  $\frac{\pi ab}{2}$  (D) None of these
80. The area bounded by the curve  $y^2 = x$  and  $x^2 = y$  is given by :  
 (A)  $0$  (B)  
 (C)  $\frac{2}{3}$  (D)  $1$
81. The whole area of the curve  $r = a \cos 2\theta$  is :  
 (A)  $\frac{\pi a^2}{2}$  (B)  $\pi a^2$   
 (C)  $2\pi a^2$  (D)  $\frac{2\pi a^2}{3}$
82. The line  $y = x + 1$  is revolved about  $x$ -axis. The volume of solid of revolution formed by revolving the area covered by the given curve,  $x$ -axis and the lines  $x = 0$ ,  $x = 2$  is :  
 (A)  $\frac{19\pi}{3}$  (B)  $\frac{17\pi}{3}$   
 (C)  $\frac{13\pi}{3}$  (D)

83. The volume generated by revolution of the ellipse about major axis is

[assume that  $a > b$ ]:

- (A)  $\frac{4\pi ab^2}{3}$  (B)  $\frac{4\pi a^2 b}{3}$   
 (C)  $\frac{4\pi a^2 b^2}{3}$  (D) None of these

84. The surface of the solid of revolution about x-axis of the area bounded by the curve  $y = x$ , x-axis and the ordinates  $x = 0$  and  $x = 3$  is equal to :

- (A)  $4\sqrt{2}\pi$  (B)  $9\sqrt{2}\pi$   
 (C)  $11\sqrt{2}\pi$  (D)  $8\sqrt{2}\pi$

85. The value of  $\int_0^{\frac{\pi}{2}} \sin^6 x \, dx =$  :

- (A)  $\frac{5\pi}{8}$  (B)  $\frac{3\pi}{8}$   
 (C)  $\frac{5\pi}{16}$  (D)  $\frac{3\pi}{16}$

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3 x \, dx = 0$   
 $\frac{3\pi}{8} + \frac{y^2}{2} = 1$

86.  $\int_0^{\pi} \cos^3 x \, dx =$

- (A) 0 (B)  $\cos\pi^3$   
 (C)  $2\cos^3\pi$  (D) Does not exist

87. Order and degree of the differential equation  $\sqrt{2\left(\frac{dy}{dx}\right)^3 + 4} = \left(\frac{d^2y}{dx^2}\right)^{3/2}$  are respectively :

- (A) order 2, degree 3 (B) order 1, degree 3  
 (C) order 3, degree 2 (D) order 3, degree 1

88. If P, Q are functions of x, then solution of differential equation  $\frac{dy}{dx} + Py = Q$  is :
- (A)  $y e^{\int P dx} = \int Q e^{\int P dx} dx + c$  (B)  $y = e^{\int P dx} \int Q e^{\int P dx} dx + C$
- (C)  $y = \int Q e^{\int P dx} dx + C$  (D) None of these
89. The differential equation of the form  $\frac{dy}{dx} + Py = Qy^n$  where P and Q are functions of x, is called :
- (A) Auxiliary equation (B) Bessel's equation
- (C) Clairaut's equation (D) Bernoulli's equation
90. The solution of  $(y \cos x + 1) dx + \sin x dy = 0$  is :
- (A)  $x - y \sin x = cx$  (B)  $y + x \sin x = c$
- (C)  $y - x \sin x = c$  (D)  $x + y \sin x = c$
91. If at every point of a certain curve the slope of the tangent equals  $\frac{-2x}{y}$ , the curve is :
- (A) A straight line (B) A parabola
- (C) A circle (D) An ellipse
92. The integrating factor for the differential equation  $(x^2y - 2xy^2) dx - (x^3 - 3x^2y) dy$  is given by :
- (A) (B)  $xy$
- (C)  $x^2y^2$  (D)  $\frac{1}{x^2y^2}$
93. The general solution of  $P = \log(px - y)$  is :
- (A)  $y = cx - e^c$  (B)  $y + cx = e^c$
- (C)  $y + x = \log c$  (D)  $y + c = e^x$
94. The general solution of a differential equation of first order represents :
- (A) A family of surfaces (B) A pair of curves in xy plane
- (C) A family of curves in xy plane (D) None of these

95. The singular solution of the differential equation  $P^3 + Px - y = 0$  is  $\left[ \text{where } P = \frac{dy}{dx} \right]$ :
- (A)  $27y^2 + 4x^3 = 0$  (B)  $y^2 = 4ax$   
 (C)  $x^2 + y^2 = a^2$  (D) None of these
96. The orthogonal trajectory of the family of curves  $ay^2 = x^3$  is :
- (A)  $3y^2 - 2x^2 = \text{constant}$  (B)  $2x^2 + y^2 = \text{constant}$   
 (C)  $3x^2 + y^2 = \text{constant}$  (D)  $2x^2 + 3y^2 = \text{constant}$
97. Solution of  $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$  is :
- (A)  $c_1e^{-2x} + c_2e^x$  (B)  $c_1e^{2x} + c_2e^x$   
 (C)  $c_1e^{2x} + c_2e^{-2x}$  (D) None of these
98. The general solution of the differential equation  $D^2(D+1)^2y = e^x$  is :
- (A)  $y = c_1 + c_2x + (c_3 + c_4x)e^x$  (B)  $y = c_1 + c_2x + (c_3 + c_4x)e^{-x} + \frac{e^x}{4}$   
 (C)  $y = c_1 + c_2e^{-x} + (c_3 + c_4x)e^{-x} + \frac{e^x}{4}$  (D) None of these
99. The particular integral of the differential equation  $(D+2)(D-1)^3y = e^x$  is :
- (A)  $\frac{x^3e^x}{18}$  (B)  $x^3e^x$   
 (C)  $\frac{x^3e^x}{3}$  (D) None of these
100. The equation of the cylinder whose generators are parallel to the line  $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$  and whose guiding curve is  $x^2 + 2y^2 = 1, z = 0$  is given by :
- (A)  $(3z-x)^2 + 2(2z+3y)^2 = 9$  (B)  $(3x+z)^2 + 2(3y-2z)^2 = 9$   
 (C)  $(3x-z)^2 + 2(3y+2z)^2 = 9$  (D)  $(2z+3x)^2 + 2(3y-x)^2 = 9$

## ROUGH WORK

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